EXTRA MATHEMATICS ADMISSIONS TEST
December 2020
Time allowed: 1 hour

Surname	
Other names	

This paper contains 10 multiple choice questions.
Calculators are not permitted.

For each question on pages 2–11 you will be given **five** possible answers, just one of which is correct. Indicate for each question **A-J** which answer (a), (b), (c), (d), or (e) you think is correct with a tick (✓) in the corresponding column in the table below.

	(a)	(b)	(c)	(d)	(e)
A					
B					
C					
D					
E					
F					
G					
H					
I					
J					

A. The distance between opposite corners of a cube is 2. The surface area of the cube equals

(a) 4, (b) 6, (c) 8, (d) 12, (e) 24.

Please turn over

B. If x is a very large positive real number, then the product

$$2^x \times 3^{-x} \times 4^x \times 5^{-x} \times \cdots \times 18^x \times 19^{-x} \times 20^x \times 21^{-x}$$

is

(a) very close to zero,

(b) slightly larger than 1,

(c) equal to 1,

(d) very close to 2,

(e) very large.

Please turn over

C. Using degrees, the number of real solutions x to the equation

$$\cos\left(\frac{240x}{x^2+4}\right) = \frac{1}{2}$$

is

(a) 0, (b) 1, (c) 2, (d) 3, (e) infinite.

Please turn over

D. Let

$$y = 2x + 3x^2 + 5x^3 + \dots$$

so that the coefficient of x^n is the n^{th} prime number. Then the value of $\dfrac{\mathrm{d}^5 y}{\mathrm{d}x^5}$ at $x = 0$ is

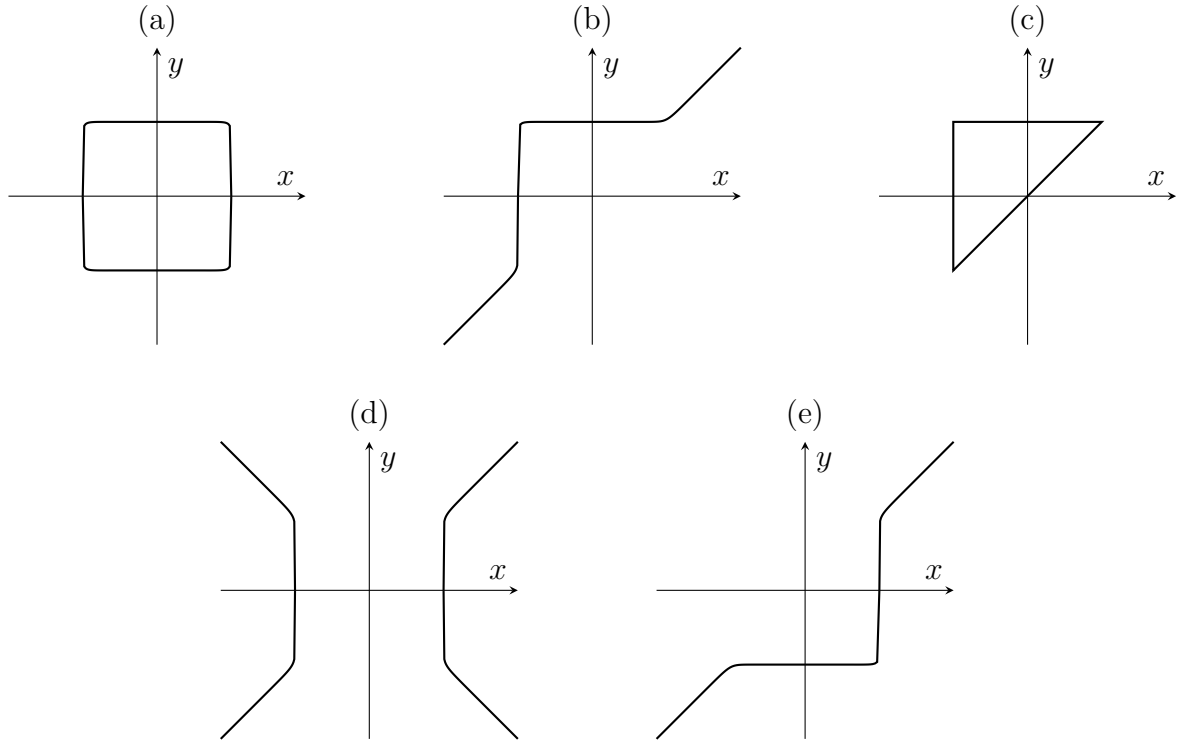
(a) 0, (b) 120, (c) 840, (d) 1080, (e) 1320.

Please turn over

E. The curve

$$x^{20} - y^{20} = 1$$

is sketched in



Please turn over

F. The following are statements about a real number x.

$$P: \quad \frac{x^2-1}{x+2} < 0, \qquad Q: \quad \frac{1+x}{1-x} > 0.$$

Then it follows that

(a) P implies Q but Q does not imply P.

(b) Q implies P but P does not imply Q.

(c) P and Q are equivalent.

(d) If P is true then Q is false.

(e) If Q is true then P is false.

Please turn over

G. The functions S and T are defined by

$$S(x) = x + 1, \quad T(x) = \frac{1}{2}x - 1.$$

Beginning with $x = 0$ the functions S and T are repeatedly applied in some order. For example, $SSTS(0) = \frac{3}{2}$. The set of possible outputs is

(a) all positive rational numbers.

(b) all rational numbers greater than -1 with denominator a power of 2 when written in lowest terms.

(c) all rational numbers with denominator a power of 2 when written in lowest terms.

(d) all positive rational numbers with denominator a power of 2 when written in lowest terms.

(e) all rational numbers greater than -2 with denominator a power of 2 when written in lowest terms.

Please turn over

H. A sequence a_n is defined by $a_0 = A$ and $a_k = (a_{k-1})^2$ for $k > 0$, where $A > 1$. The sequence b_n is defined by $b_n = \log_2 a_n$. The sequence b_n is

(a) constant

(b) an arithmetic progression

(c) a geometric progression

(d) all of the above

(e) none of the above

Please turn over

I. An equilateral triangle is drawn in the xy-plane. Two of its vertices are at $(0, 0)$ and $(1000, 0)$. The number of points (x, y) *inside* the triangle, where x and y are both whole numbers, equals

(a) 866,025, (b) 866,026, (c) 866,027, (d) 432,512, (e) 432,513.

[Note that $\sqrt{3} = 1.7321$ to 4 decimal places.]

Please turn over

J. Let R be the region where all four of the following inequalities hold

$$x^2 < 2 + y, \quad x^2 < 2 - y, \quad y^2 < 2 + x, \quad y^2 < 2 - x.$$

What is the area of R?

(a) 0, (b) $\frac{28}{3}$, (c) $4 + 2\pi$, (d) $\frac{4}{3}\left(8\sqrt{2} - 7\right)$, (e) infinite.

End of last question